# The gravitational field in the relativistic uniform model within the framework of the covariant theory of gravitation

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30 June 2016

**Abstract**

The relativistic theory describes the physics of phenomena more precisely than classical mechanics. This leads to the fact that an ideal uniform model of a body with a constant mass density must be replaced by the relativistic uniform model. In the relativistic model the mass density can be the coordinate function, but it is considered a constant invariant mass density in the reference frames, associated with the particles that make up the body. Due to the motion of the particles the effective mass density in the system differs from the invariant values, which introduces additional corrections into the values of the field functions and into the system's energy. For the relativistic uniform system with an invariant mass density the exact expressions are determined for the potentials and strengths of the gravitational field, the energy of particles and fields. It is shown that, as in the classical case for bodies with a constant mass density, in the system with a zero vector potential of the gravitational field, the energy of the particles, associated with the scalar field potential, is twice as large in the absolute value as the energy defined by the tensor invariant of the gravitational field.

# 1 Introduction

Various properties of the relativistic uniform system were discussed earlier in [1–3]. The purpose of this paper is to define more precisely the results relating to the gravitational field in the framework of the covariant theory of gravitation, to calculate the second-order corrections and to verify the relations between the energy of the particles in the scalar gravitational potential and the proper energy of the gravitational field.

# 2 The field functions

As a uniform relativistic system the spherical system is considered, which consists of the particles that can also have the electrical charge. The stability of the system is maintained by the action of its proper gravitation, the internal pressure field and the acceleration field of the particles [4, 5]. The field functions are calculated on the assumption that there is no general rotation of the particles in the system, they move randomly and therefore the total vector field potentials on the average tend to zero. The equation for the gravitational scalar potential inside the sphere and its solution in the weak field limit have the following form [2]:

$$\triangle\psi_i = 4\pi G\rho_0\gamma', \tag{1}$$

$$\psi_i = \frac{Gc^2\gamma_c}{\eta}\cos\left(\frac{a}{c}\sqrt{4\pi\eta\rho_0}\right) - \frac{Gc^3\gamma_c}{r\eta\sqrt{4\pi\eta\rho_0}}\sin\left(\frac{r}{c}\sqrt{4\pi\eta\rho_0}\right) \approx \frac{2\pi G\rho_0\gamma_c(r^2-3a^2)}{3}.$$

In (1) the Lorentz factor of the particles is $\gamma' = \frac{1}{\sqrt{1-v'^2/c^2}}$ , $\mathbf{v}'$ is the average velocity of an arbitrary particle inside the sphere, $c$ is the speed of light, $G$ is the gravitational constant, $\rho_0$ is the mass density of the particle in the reference frame associated with the particle, the index $i$ differentiates the internal gravitational scalar potential $\psi_i$ from external potential $\psi_o$ , which is generated by the sphere outside its limits. Both potential $\psi_i$ and $\gamma'$ are the functions of the current radius $r$ inside the sphere and do not depend on the angular variables.

The dependence of $\gamma'$ on the radius was found in [1]:

$$\gamma' = \frac{c\gamma_c}{r\sqrt{4\pi\eta\rho_0}}\sin\left(\frac{r}{c}\sqrt{4\pi\eta\rho_0}\right) \approx \gamma_c - \frac{2\pi\eta\rho_0 r^2\gamma_c}{3c^2}, \tag{2}$$

where $\gamma_c$ is the Lorentz factor of the particles at the center of the sphere, $\eta$ is the acceleration field coefficient.

For the external gravitational potential $\psi_o$ of the fixed sphere, filled with moving particles, we obtain the following:

$$\psi_o = -\frac{Gc^3\gamma_c}{r\eta\sqrt{4\pi\eta\rho_0}}\left[\sin\left(\frac{a}{c}\sqrt{4\pi\eta\rho_0}\right) - \frac{a}{c}\sqrt{4\pi\eta\rho_0}\cos\left(\frac{a}{c}\sqrt{4\pi\eta\rho_0}\right)\right]. \tag{3}$$

The mass $m$ is defined as the product of the mass density $\rho_0$ by the sphere's volume $V_s$ . However, the actual gravitational field outside the sphere is defined by the mass $m_b$ , which is equal to:

$$m_b = \rho_0 \int \gamma' \, dV_s = \frac{c^3 \gamma_c}{\eta \sqrt{4\pi\eta\rho_0}} \left[ \sin\left( \frac{a}{c} \sqrt{4\pi\eta\rho_0} \right) - \frac{a}{c} \sqrt{4\pi\eta\rho_0} \cos\left( \frac{a}{c} \sqrt{4\pi\eta\rho_0} \right) \right] \approx$$
$$\approx m\gamma_c \left( 1 - \frac{3\eta m}{10ac^2} \right).$$

The mass $m_b$ represents the sum of the invariant masses of all the particles in the system, which is equal to the gravitational mass of the system $m_g$ . In view of the definition of the mass $m_b$ , from (3) it follows:

$$\psi_o = -\frac{Gm_b}{r} \approx -\frac{Gm\gamma_c}{r} \left( 1 - \frac{3\eta m}{10ac^2} \right).$$

Since after averaging over a sufficient number of particles, the internal vector gravitational potential $\mathbf{D}_i$ and the external vector gravitational potential $\mathbf{D}_o$ of the system are equal to zero, the acting gravitational field strengths inside and outside the system are actually defined only by the gradient of the corresponding scalar potential. In view of (1) and (3), for the strengths we obtain the following:

$$\mathbf{\Gamma}_i = -\nabla\psi_i - \frac{\partial \mathbf{D}_i}{\partial t} = -\frac{Gc^3\gamma_c \mathbf{r}}{r^3 \eta \sqrt{4\pi\eta\rho_0}} \left[ \sin\left( \frac{r}{c} \sqrt{4\pi\eta\rho_0} \right) - \frac{r}{c} \sqrt{4\pi\eta\rho_0} \cos\left( \frac{r}{c} \sqrt{4\pi\eta\rho_0} \right) \right] \approx$$
$$\approx -\frac{4\pi G\rho_0 \gamma_c \mathbf{r}}{3} \left( 1 - \frac{4\pi\eta\rho_0 r^2}{10c^2} \right). \tag{4}$$

$$\mathbf{\Gamma}_o = -\nabla\psi_o - \frac{\partial \mathbf{D}_o}{\partial t} = -\frac{Gc^3\gamma_c \mathbf{r}}{r^3 \eta \sqrt{4\pi\eta\rho_0}} \left[ \sin\left( \frac{a}{c} \sqrt{4\pi\eta\rho_0} \right) - \frac{a}{c} \sqrt{4\pi\eta\rho_0} \cos\left( \frac{a}{c} \sqrt{4\pi\eta\rho_0} \right) \right] =$$
$$= -\frac{Gm_b \mathbf{r}}{r^3} \approx -\frac{Gm\gamma_c \mathbf{r}}{r^3} \left( 1 - \frac{3\eta m}{10ac^2} \right). \tag{5}$$

The torsion field, which has the same meaning in the covariant theory of gravitation as the gravitomagnetic field in gravitomagnetism, on the average is equal to zero, both inside and outside of the system under consideration:

$$\mathbf{\Omega}_i = \nabla \times \mathbf{D}_i = 0, \qquad \mathbf{\Omega}_o = \nabla \times \mathbf{D}_o = 0. \tag{6}$$

## 3 The energy of the particles in the field and the energy of the field itself

We will calculate the energy of the particles in the gravitational field of the system, in which the vector potential and the torsion field on the average are equal to zero. The energy of the particles in this case is defined as the volume integral taken of the product of the effective mass density inside the sphere $\rho = \rho_0 \gamma'$ by the internal scalar potential $\psi_i$ . In view of (1), (2) we obtain the following:

$$\int \rho_0 \psi_i \gamma' \, dV = \frac{G c^4 \gamma_c^2}{\eta^2} \left[ \frac{c}{\sqrt{4\pi\eta\rho_0}} \sin\left(\frac{a}{c}\sqrt{4\pi\eta\rho_0}\right) - a\cos\left(\frac{a}{c}\sqrt{4\pi\eta\rho_0}\right) \right] \cos\left(\frac{a}{c}\sqrt{4\pi\eta\rho_0}\right) - \\ - \frac{G c^4 \gamma_c^2}{\eta^2} \left\{ \frac{a}{2} - \frac{c}{4\sqrt{4\pi\eta\rho_0}} \sin\left(\frac{2a}{c}\sqrt{4\pi\eta\rho_0}\right) \right\} \approx -\frac{6 G m^2 \gamma_c^2}{5a} \left(1 - \frac{4\eta m}{7ac^2}\right). \quad (7)$$

We will now calculate the volume integral taken of the tensor invariant of the gravitational field, separately for the field inside and outside the sphere. The integral of the tensor invariant is expressed in terms of the gravitational field strength and the torsion field:

$$-\int \frac{c^2}{16\pi G} \Phi_{\mu\nu} \Phi^{\mu\nu} \, dV = \frac{1}{8\pi G} \int \left(\Gamma^2 - c^2 \Omega^2\right) dV.$$

This integral part is included in this form in the Hamiltonian of the system and defines there the contribution of the gravitational field. Substituting here (4), (5), (6), we find:

$$-\int_{r=0}^{a} \frac{c^2}{16\pi G} \Phi_{\mu\nu} \Phi^{\mu\nu} \, dV = \\ = \frac{G c^4 \gamma_c^2}{2\eta^2} \left[ \frac{a}{2} + \frac{c}{4\sqrt{4\pi\eta\rho_0}} \sin\left(\frac{2a}{c}\sqrt{4\pi\eta\rho_0}\right) - \frac{c^2}{4\pi\eta\rho_0 a} \sin^2\left(\frac{a}{c}\sqrt{4\pi\eta\rho_0}\right) \right] \approx \\ \approx \frac{G m^2 \gamma_c^2}{10a} \left(1 - \frac{3\eta m}{7ac^2}\right). \quad (8)$$

$$-\int_{r=a}^{\infty} \frac{c^2}{16\pi G} \Phi_{\mu\nu} \Phi^{\mu\nu} \, dV = \frac{G c^4 \gamma_c^2}{2\eta^2 a} \left[ \frac{c}{\sqrt{4\pi\eta\rho_0}} \sin\left(\frac{a}{c}\sqrt{4\pi\eta\rho_0}\right) - a\cos\left(\frac{a}{c}\sqrt{4\pi\eta\rho_0}\right) \right]^2 \approx \\ \approx \frac{G m^2 \gamma_c^2}{2a} \left(1 - \frac{3\eta m}{5ac^2}\right).$$

# 4 Conclusions

One of the conclusions in [6] was that the energy of the motionless matter of the uniform body in the form of a sphere in its proper static gravitational field is twice as large in its absolute value as the energy of the gravitational field itself. What will happen, if we turn into the relativistic uniform system, in which the matter particles are not motionless, but are moving with the Lorentz factor (2), depending on the current radius? To answer this question we must sum up the integrals in (8), that is, calculate the integral of the tensor invariant over the entire volume, occupied by the field, and then compare the result with (7). We obtain the following:

$$-\int_{r=0}^{a} \frac{c^2}{16\pi G} \Phi_{\mu\nu} \Phi^{\mu\nu} \, dV - \int_{r=a}^{\infty} \frac{c^2}{16\pi G} \Phi_{\mu\nu} \Phi^{\mu\nu} \, dV = -\int_{r=0}^{\infty} \frac{c^2}{16\pi G} \Phi_{\mu\nu} \Phi^{\mu\nu} \, dV = -\frac{1}{2} \int \rho_0 \psi_i \gamma' \, dV.$$

Hence we see that both in the classical case and in the relativistic case, the relation between the energy of the particles in the field and the energy of the field itself remains unchanged.

# References


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